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Entropy and supersymmetry of D dimensional extremal electric black holes versus string states

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Following the work of Sen, we consider the correspondence between extremal black holes and string states in the context of the entropy. We obtain and study properties of electrically charged black hole backgrounds of tree level heterotic string theory compactified on a p dimensional torus, for $D = (10 - p) = 4 \dots 9$. We study in particular a one-parameter extremal class of these black holes, the members of which are shown to be supersymmetric. We find that the entropy of such an extremal black hole, when calculated at the stringy stretched horizon, scales in such a way that it can be identified with the entropy of the elementary string state with the corresponding quantum numbers.

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Introduction

Recently there has been considerable interest in the question of whether black hole configurations of string theory can or must be identified as string states. The question has arisen in studies of duality, of the configurations themselves, and also in the study of the black hole information problem [1, 2, 3, 4, 5, 6, 7, 8] .

Qualitatively, the idea of identifying string states as black holes is a simple one. At a given value of the string coupling, most states above a certain mass level have a Compton wavelength which is shorter than their Schwarzschild radius, and so may be considered to be black holes. However, there are some string states which should not be identified as black holes, because they rotate too fast for their effective radius to be inside their Schwarzschild radius.

The question of whether nonextremal uncharged black holes could be identified as string states was considered in the context of the entropy in [1, 3]. The entropy for the black holes was obtained using the usual formula involving the area of the event horizon in the Einstein metric, and for the string states it was simply the logarithm of the degeneracy of the string states with a given mass (and angular momentum). It was found that the scaling permitted an identification of black holes as string states, as long as the mass used for the black hole was the Rindler mass at the stretched horizon[1]. This Rindler mass is related to the ADM mass by the redshift between the stretched horizon and asymptotic infinity, which is a large number of order the mass of the black hole. The stretched horizon, located at a proper distance of order one string unit away from the event horizon, is the place where stringy effects start coming into play which are likely to be important for the resolution of the black hole information problem [9, 10, 11].

For extremal black holes, a comparison with string states was made in the context of heterotic string theory compactified on a six dimensional torus[4]. It was found that the quantum numbers of several extremal black hole solutions matched those of supersymmetric string states of the same theory. Then in a very interesting work[8], it was found that the entropy of extremal electrically charged black holes, when calculated at the stringy stretched horizon, scaled in such a way that it agreed with the string entropy obtained by taking the logarithm of the degeneracy of string states with corresponding quantum numbers. It was necessary in this case to calculate the entropy at the stringy stretched horizon: classically the area of the event horizon of these extremal black holes is zero, but this is not expected to survive quantum corrections. For extremal black holes there is no redshift-induced renormalization of the mass, and so the agreement of the entropies was striking.

In [2] a concern had been voiced that the scaling of the entropy for black holes in D di-

mensions might preclude an identification with string states. However, it was seen in the abovementioned works that this turned out not to be a problem in $D = 4$, provided that one calculated the relevant quantities at the stretched horizon.

In this work we will study electrically charged black hole backgrounds of tree level heterotic string theory compactified on a p dimensional torus, where $D = (10 - p) = 4 \dots 9$, with a view to understanding whether the correspondence found in [8] is specific to $D = 4$. Along the way, we study supersymmetry, which is expected[12] to exist for the extremal black holes in $D = 4$ which have a fixed relation between ADM mass and right-handed charge.

The plan of the paper is as follows. In Section 1 we review the solution-generating techniques which will be used in Section 2 to obtain the D dimensional electrically charged black holes. Supersymmetry of a one-parameter extremal “type \mathcal{R} ” class of these black holes is studied in Section 3, where it is found that half of the possible supersymmetries are unbroken. Section 4 contains a discussion of the entropy and the stretched horizon of these black holes. We find that the entropy of type \mathcal{R} black holes, when calculated at the stringy stretched horizon, agrees with the entropy of elementary string states of the same quantum numbers. We end with some conclusions; our notation and conventions are listed in the appendix.

1 Actions and solution-generating

In order to obtain general spherically symmetric electrically charged black hole backgrounds of the tree level action for heterotic string theory compactified on a p dimensional torus, we will employ solution-generating techniques. We will begin by reviewing the tree level actions in D and ten dimensions, and the solution-generating methods[13].

The massless bosonic fields in heterotic string theory compactified on a p dimensional torus are the metric tensor $G_{\mu\nu}$, the anti-symmetric tensor field $B_{\mu\nu}$, $k = (16 + 2p)$ $U(1)$ gauge fields A_μ^α ($1 \leq \alpha \leq k$), the scalar dilaton field Φ , and a $k \times k$ matrix-valued scalar moduli field M satisfying

$$MLM^T = L \quad M^T = M. \quad (1.1)$$

In this equation L is a $k \times k$ symmetric matrix with $(16 + p)$ eigenvalues -1 and p eigenvalues $+1$. We will take L to be

$$L = \begin{pmatrix} -\mathbf{1}_{16+p} & \\ & \mathbf{1}_p \end{pmatrix} \quad (1.2)$$

where $\mathbf{1}_n$ denotes the $n \times n$ identity matrix.

The action describing the effective field theory of these fields is [14, 15]

$$\begin{aligned}
S = \int d^D x \sqrt{-G} e^{-\Phi} & \left[R[G] + G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} G^{\mu\nu} \text{Tr}(\partial_\mu M L \partial_\nu M L) \right. \\
& \left. - \frac{1}{12} G^{\mu\lambda} G^{\nu\kappa} G^{\rho\sigma} H_{\mu\nu\rho} H_{\lambda\kappa\sigma} - \frac{1}{4} G^{\mu\lambda} G^{\nu\kappa} F_{\mu\nu}^\alpha (L M L)_{\alpha\beta} F_{\lambda\kappa}^\beta \right]
\end{aligned} \tag{1.3}$$

where

$$\begin{aligned}
F_{\mu\nu}^\alpha &= \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha \\
H_{\mu\nu\rho} &= \partial_\mu B_{\nu\rho} + \frac{1}{2} A_\mu^\alpha L_{\alpha\beta} F_{\nu\rho}^\beta + (\text{cyclic})
\end{aligned} \tag{1.4}$$

and $R[G]$ is the scalar curvature formed from the metric $G_{\mu\nu}$.

We note that it is $e^{-\Phi}$ which appears in front of the action in string metric (1.3), so we have for the string coupling at asymptotic infinity

$$g_\infty = \langle e^{\Phi/2} \rangle_\infty \quad . \tag{1.5}$$

The action (1.3) is invariant under the $O(16+p, p)$ transformation

$$M \rightarrow \Omega M \Omega^T \quad A_\mu \rightarrow \Omega A_\mu \quad \Phi \rightarrow \Phi \quad G_{\mu\nu} \rightarrow G_{\mu\nu} \quad B_{\mu\nu} \rightarrow B_{\mu\nu} \tag{1.6}$$

where Ω is a $k \times k$ matrix satisfying

$$\Omega L \Omega^T = L \quad . \tag{1.7}$$

This is a symmetry of the full string theory if the k dimensional lattice Λ of electric charges is also rotated by Ω [16]. For fixed Λ , only a discrete subgroup $O(16+p, p; \mathbb{Z})$ is a symmetry.

For time-independent backgrounds, it is expected that there is an enlarged symmetry, namely $O(17+p, 1+p)$ [17]. In the barred variables, defined by

$$\begin{aligned}
\bar{A}_i^\alpha &= A_i^\alpha - (G_{tt})^{-1} G_{ti} A_t^\alpha \quad 1 \leq \alpha \leq k \quad 1 \leq i \leq (D-1) \\
\bar{A}_i^{(k+1)} &= (G_{tt})^{-1} G_{ti} \\
\bar{A}_i^{(k+2)} &= B_{it} + \frac{1}{2} A_t^\alpha L_{\alpha\beta} \bar{A}_i^\beta \\
\bar{G}_{ij} &= G_{ij} - (G_{tt})^{-1} G_{ti} G_{tj} \\
\bar{B}_{ij} &= B_{ij} + \bar{A}_{[i}^{(k+1)} \bar{A}_{j]}^{(k+2)} \\
\bar{\Phi} &= \Phi - \frac{1}{2} \ln(-G_{tt})
\end{aligned} \tag{1.8}$$

and

$$\bar{M}^{-1} = \begin{pmatrix} M + (G_{tt})^{-1} A_t A_t^T & -(G_{tt})^{-1} A_t & M L A_t + (G_{tt})^{-1} A_t (A_t^T L A_t) \\ -(G_{tt})^{-1} A_t^T & (G_{tt})^{-1} & -(G_{tt})^{-1} A_t^T L A_t \\ A_t^T L M & -(G_{tt})^{-1} A_t^T L A_t & G_{tt} + A_t^T L M L A_t \\ +(G_{tt})^{-1} A_t^T (A_t^T L A_t) & & +(G_{tt})^{-1} (A_t^T L A_t)^2 \end{pmatrix} \quad (1.9)$$

and

$$\bar{L} = \begin{pmatrix} L & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.10)$$

the action can be rewritten as

$$\begin{aligned} S = C \int dt \int d^{D-1} x \sqrt{\bar{G}} e^{-\bar{\Phi}} & \left[R_{\bar{G}} + \bar{G}^{ij} \partial_i \bar{\Phi} \partial_j \bar{\Phi} + \frac{1}{8} \bar{G}^{ij} \text{Tr}(\partial_i \bar{M} \bar{L} \partial_j \bar{M} \bar{L}) \right. \\ & \left. - \frac{1}{12} \bar{G}^{il} \bar{G}^{jm} \bar{G}^{kn} \bar{H}_{ijk} \bar{H}_{lmn} - \frac{1}{4} \bar{G}^{ik} \bar{G}^{jl} \bar{F}_{ij}^{\bar{\alpha}} (\bar{L} \bar{M} \bar{L})_{\bar{\alpha}\bar{\beta}} \bar{F}_{kl}^{\bar{\beta}} \right] \end{aligned} \quad (1.11)$$

where

$$\begin{aligned} \bar{F}_{ij}^{\bar{\alpha}} &= \partial_i \bar{A}_j^{\bar{\alpha}} - \partial_j \bar{A}_i^{\bar{\alpha}}, \quad 1 \leq \bar{\alpha} \leq k+2, \\ \bar{H}_{ijk} &= \partial_i \bar{B}_{jk} + \frac{1}{2} \bar{A}_i^{\bar{\alpha}} \bar{L}_{\bar{\alpha}\bar{\beta}} \bar{F}_{jk}^{\bar{\beta}} + (\text{cyclic}) \quad . \end{aligned} \quad (1.12)$$

The $O(17+p, 1+p)$ symmetry then acts as

$$\bar{M} \rightarrow \bar{\Omega} \bar{M} \bar{\Omega}^T \quad \bar{A}_i \rightarrow \bar{\Omega} \bar{A}_i \quad \bar{\Phi} \rightarrow \bar{\Phi} \quad \bar{G}_{ij} \rightarrow \bar{G}_{ij} \quad \bar{B}_{ij} \rightarrow \bar{B}_{ij} \quad (1.13)$$

where $\bar{\Omega}$ is a $(k+2) \times (k+2)$ matrix satisfying

$$\bar{\Omega} \bar{L} \bar{\Omega}^T = \bar{L} \quad . \quad (1.14)$$

We will work with the parametrization of $\bar{\Omega}$ where \bar{L} is diagonal; the orthogonal matrix U that diagonalizes \bar{L} is given by

$$U = \begin{pmatrix} \mathbf{1}_k & & \\ & \frac{+1}{\sqrt{2}} & \frac{+1}{\sqrt{2}} \\ & \frac{-1}{\sqrt{2}} & \frac{+1}{\sqrt{2}} \end{pmatrix} \quad (1.15)$$

so that

$$U \bar{L} U^T \equiv \bar{L}_d = \text{diag}(-\mathbf{1}_{16+p}, \mathbf{1}_p, 1, -1) \quad . \quad (1.16)$$

Then $U\bar{\Omega}U^T$ preserves \bar{L}_d .

As was done in [12], we will apply this $O(17+p, 1+p)$ transformation to a known time-independent classical black hole solution to generate more classical solutions of the equations of motion. Without loss of generality, we can restrict to solutions with fields which tend to zero at asymptotic infinity – except for the metric, which tends to the Minkowski metric, and the moduli matrix which tends to the unit matrix. With these conditions, we use only the subgroup $O(16+p, 1) \otimes O(p, 1)$; the rest of the group generates pure gauge deformations[17]. The solution which we start with will be uncharged and will have a trivial (unit) moduli matrix, so we see by looking at (1.8)–(1.9) that we will need to mod out by an $O(16+p) \otimes O(p)$ symmetry. Thus the transformations which generate inequivalent solutions live in the coset space

$$[O(16+p, 1) \otimes O(p, 1)] / [(O(16+p) \otimes O(p))] \quad (1.17)$$

which has dimension $(18+2p) = (k+2)$. We will use the following parametrization [12] of a generic element:

$$\bar{\Omega} = \bar{\Omega}_2 \bar{\Omega}_1 \quad (1.18)$$

where

$$U\bar{\Omega}_1 U^T = \begin{pmatrix} \mathbf{1}_{15+p} & 0 & 0 & 0 & 0 & 0 \\ 0 & \cosh \alpha & 0 & 0 & \sinh \alpha & 0 \\ 0 & 0 & \mathbf{1}_{p-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \cosh \beta & 0 & \sinh \beta \\ 0 & \sinh \alpha & 0 & 0 & \cosh \alpha & 0 \\ 0 & 0 & 0 & \sinh \beta & 0 & \cosh \beta \end{pmatrix} \quad (1.19)$$

and

$$\bar{\Omega}_2 = \begin{pmatrix} R_{16+p}(\vec{n}_L) & & \\ & R_p(\vec{n}_R) & \\ & & \mathbf{1}_2 \end{pmatrix} \quad (1.20)$$

where

$$R_N(\vec{n}) \begin{pmatrix} \vec{0}_{N-1} \\ 1 \end{pmatrix} = \vec{n} \quad . \quad (1.21)$$

Here \vec{n}_L and \vec{n}_R are arbitrary $(16+p)$ and p dimensional unit vectors respectively. So $\bar{\Omega}$ in the above equations is parametrized by $(k+2)$ parameters: α, β, \vec{n}_L and \vec{n}_R . $\alpha, \beta \in [0, \infty)$ are the boost parameters of the noncompact group we use to generate solutions; α is the parameter of the left-handed part of the group and β is the parameter of the right-handed part of the group.

We now turn to the question of how ten and D dimensional fields are related. The relationship between the vector and moduli fields and the ten dimensional fields is a little involved [14].

The first step of the dictionary is a change of basis. In [14, 18] the matrix L is chosen to be off-diagonal, and in order to take the moduli matrix and vector fields to the ones appropriate to the off-diagonal L , the matrix

$$U_\star = \begin{pmatrix} \mathbf{1}_{16} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}\mathbf{1}_p & \frac{-1}{\sqrt{2}}\mathbf{1}_p \\ 0 & \frac{1}{\sqrt{2}}\mathbf{1}_p & \frac{1}{\sqrt{2}}\mathbf{1}_p \end{pmatrix} \quad (1.22)$$

must be used to transform A and M as

$$A \rightarrow A_\star = U_\star^T A \quad M \rightarrow M_\star = U_\star^T M U_\star \quad (1.23)$$

where $A = \begin{pmatrix} A_L \\ A_R \end{pmatrix}$.

Once this change of basis has been done, and with a split of the left-handed directions into 16 and p dimensional parts,

$$M_\star = \begin{pmatrix} \mathbf{1}_{16} + aG^{-1}a^T & aG^{-1}C + a & aG^{-1} \\ C^T G^{-1}a^T + a^T & G + C^T G^{-1}C + a^T a & C^T G^{-1} \\ G^{-1}a^T & G^{-1}C & G^{-1} \end{pmatrix} \quad A_\star = \begin{pmatrix} A^{[3]} \\ A^{[2]} \\ A^{[1]} \end{pmatrix} \quad (1.24)$$

where $C = B + \frac{1}{2}a^T a$, $B = (B_{\alpha\beta})$, $a = (a_\alpha^I)$, etc. Then (upon doing a sign change on the left-handed A_\star fields which is necessary[19] to convert them to the signature convention of [14]) we have for the ten dimensional fields[14], denoted by tildes,

$$\begin{aligned} \tilde{A}_\alpha^I &= a_\alpha^I \\ \tilde{A}_\mu^I &= a_\alpha^I A_\mu^{[1]\alpha} + A_\mu^{[3]I} \\ \tilde{B}_{\alpha\beta} &= B_{\alpha\beta} \\ \tilde{B}_{t\alpha} &= A_{t\alpha}^{[2]} - B_{\alpha\beta} A_t^{[1]\beta} - \frac{1}{2}a_\alpha^I A_t^{[3]I} \\ \tilde{G}_{\alpha\beta} &= G_{\alpha\beta} \\ \tilde{E} &= \begin{pmatrix} E_\mu^m & A_\mu^{[1]\alpha} E_\alpha^a \\ 0 & E_\alpha^a \end{pmatrix} \\ \tilde{E}^{-1} &= \begin{pmatrix} E_m^\mu & -A_\mu^{[1]\alpha} E_m^\mu \\ 0 & E_a^\alpha \end{pmatrix} \\ \tilde{\omega}[\tilde{G}]_{0,0i} &= -\partial_i \Phi \quad \tilde{\omega}[\tilde{G}]_{0,ia} = \frac{1}{2}F_{ti\alpha}^{[1]} E_a^\alpha E_0^t \end{aligned}$$

$$\begin{aligned}
\tilde{\omega}[\tilde{G}]_{i,ab} &= \frac{1}{2}E_a^\alpha \partial_i E_{\alpha b} - (a \leftrightarrow b) & \tilde{\omega}[\tilde{G}]_{i,0a} &= \frac{-1}{2}F_{ti\alpha}^{[1]}E_a^\alpha E_0^t \\
\tilde{\omega}[\tilde{G}]_{a,0i} &= \frac{-1}{2}F_{ti}^{[1]\alpha}E_0^t E_{\alpha a} & \tilde{\omega}[\tilde{G}]_{a,ib} &= \frac{-1}{2}E_a^\alpha E_b^\beta \partial_i G_{\alpha\beta} \\
\tilde{\Phi} &= \Phi + \log \sqrt{\det(G_{\alpha\beta})}
\end{aligned} \tag{1.25}$$

where $\tilde{\omega}$ are the spin-connections[20]. We have not listed $\tilde{B}_{\mu\nu}, B_{\mu\nu}$ because they will be zero for the black holes which we study. The above ten versus D dimensional field relations will be useful in Section 3 where we consider supersymmetry. The action of the ten dimensional fields is then

$$\begin{aligned}
S_{10} &= \int d^{10}x \sqrt{-\tilde{G}} e^{-\tilde{\Phi}} \left[R[\tilde{G}] + \tilde{G}^{\tilde{\mu}\tilde{\nu}} \partial_{\tilde{\mu}} \tilde{\Phi} \partial_{\tilde{\nu}} \tilde{\Phi} - \frac{1}{12} \tilde{G}^{\tilde{\mu}\tilde{\tau}} \tilde{G}^{\tilde{\nu}\tilde{\sigma}} \tilde{G}^{\tilde{\rho}\tilde{\kappa}} \tilde{H}_{\tilde{\mu}\tilde{\nu}\tilde{\rho}} \tilde{H}_{\tilde{\tau}\tilde{\sigma}\tilde{\kappa}} \right. \\
&\quad \left. - \frac{1}{4} \tilde{G}^{\tilde{\mu}\tilde{\tau}} \tilde{G}^{\tilde{\nu}\tilde{\sigma}} \tilde{F}_{\tilde{\mu}\tilde{\nu}} \tilde{F}_{\tilde{\tau}\tilde{\sigma}} \right] .
\end{aligned} \tag{1.26}$$

We end this Section by giving the relationship between string and Einstein variables. In $D = (10 - p)$ dimensions, the relation between the string metric G and the Einstein metric g is

$$G_{\mu\nu} = e^{\gamma\Phi} g_{\mu\nu} \tag{1.27}$$

where we have defined the fraction γ to be

$$\gamma = \frac{2}{(D-2)} . \tag{1.28}$$

In four dimensions $\gamma = 1$. In the Einstein metric, the D dimensional action may then be expressed as

$$\begin{aligned}
S[g] &= \int d^Dx \sqrt{-g} \left[R[g] - \frac{1}{(D-2)} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} g^{\mu\nu} \text{Tr}(\partial_\mu M L \partial_\nu M L) \right. \\
&\quad \left. - \frac{1}{12} e^{-2\gamma\Phi} g^{\mu\lambda} g^{\nu\kappa} g^{\rho\sigma} H_{\mu\nu\rho} H_{\lambda\kappa\sigma} - \frac{1}{4} e^{-\gamma\Phi} g^{\mu\lambda} g^{\nu\kappa} F_{\mu\nu}^\alpha (L M L)_{\alpha\beta} F_{\lambda\kappa}^\beta \right]
\end{aligned} \tag{1.29}$$

2 The black holes

In this Section we will follow [12] and apply the solution-generating transformations of the previous Section in order to obtain electrically charged black hole backgrounds.

Let us look back to the equations for the barred variables in terms of the unbarred ones (1.8)–(1.9). It is apparent that, in order to generate a background with electric charges

from an uncharged one, we need to begin with a black hole which has nonzero angular momentum (and thus a nonzero $G_{t\varphi}$). The higher-dimensional analogue of the Schwarzschild solution is therefore insufficient for our purposes. Our starting point will instead be the higher-dimensional analogue of the Kerr solution, which has mass M and just one² rotation parameter a . The metric is valid for $D > 3$ and, in the analogue of Boyer–Lindquist coordinates, it is[21]

$$\begin{aligned} d\hat{S}^2 &= d\hat{s}^2 \\ &= -dt^2 + \sin^2 \theta (r^2 + a^2) d\varphi^2 + \Delta (dt + a \sin^2 \theta d\varphi)^2 \\ &\quad + \Psi dr^2 + \sigma^2 d\theta^2 + r^2 \cos^2 \theta d\Omega_{D-2}^2 \end{aligned} \quad (2.1)$$

where

$$\Delta = \frac{\hat{\mu}}{r^{D-5}\sigma^2} \quad \Psi = \frac{r^{D-5}\sigma^2}{r^{D-5}(r^2 + a^2) - \hat{\mu}} \quad \sigma^2 = r^2 + a^2 \cos^2 \theta \quad . \quad (2.2)$$

We have used hatted variables to distinguish this solution from the one which we are about to obtain. Notice also that the line elements in the string and Einstein metrics, $d\hat{S}^2$ and $d\hat{s}^2$ respectively, are equal because there is no dilaton for the rotating black hole; there are no gauge fields either. In the above, the parameter $\hat{\mu}$ is related to the ADM mass by

$$\hat{M}_{ADM} = \frac{(D-2)A_{D-2}}{16\pi G_N} \hat{\mu} \quad (2.3)$$

where $A_{D-2} = 2\pi^{\frac{D-1}{2}}/\Gamma(\frac{D-1}{2})$ is the area of a unit sphere in $(D-2)$ dimensions, and G_N is the Newton constant which we will hold fixed throughout.

In order to begin to form the charged background, we need the barred variables (1.8)–(1.9). The nonzero quantities are

$$\begin{aligned} \bar{\hat{A}}_{\varphi}^{(k+1)} &= \frac{\Delta a}{(\Delta - 1)} \sin^2 \theta \\ \bar{\hat{M}} &= \text{diag}(\mathbf{1}, \frac{1}{\Delta - 1}, \Delta - 1) \\ \bar{\hat{\Phi}} &= -\frac{1}{2} \log(1 - \Delta) \\ \bar{\hat{G}}_{ij} &= \hat{G}_{ij} \\ \bar{\hat{G}}_{\varphi\varphi} &= \sin^2 \theta \left[(r^2 + a^2) - \frac{\Delta a^2 \sin^2 \theta}{(\Delta - 1)} \right] \quad . \end{aligned} \quad (2.4)$$

²In D dimensions there are $[\frac{D-1}{2}]$ rotational Casimirs for a massive rep. of the Poincaré group.

After transforming the barred-hatted variables via (1.13) to the barred variables denoting the charged background, and untangling some of the relations (1.8)–(1.9) between barred and unbarred variables, we obtain

$$\begin{aligned} G_{tt}^{-1} &= -1 + (\cosh \alpha \cosh \beta) f^- + \frac{1}{2}(\cosh^2 \alpha + \cosh^2 \beta)(1 + f^+) \\ f^\pm &= \frac{1}{2}(\Delta - 1 \pm \frac{1}{\Delta - 1}) \quad . \end{aligned} \quad (2.5)$$

For the remainder of this work, we will be interested in studying possible relations between nonrotating extremal black holes and elementary string states. We therefore set the rotation parameter a to zero now. This could not have been done before doing the boosting operation, because we would not have been able to generate the full complement of charged backgrounds.

After untangling of the remaining relations, the fields of the charged black hole are found to be

$$\begin{aligned} e^{-2\Phi} &= 1 + \frac{\hat{\mu}}{\rho}(\cosh \alpha \cosh \beta - 1) + \frac{\hat{\mu}^2}{4\rho^2}(\cosh \alpha - \cosh \beta)^2 \\ dS^2 &= \frac{-(\rho^2 - \rho\hat{\mu})dt^2}{[\rho^2 + \rho\hat{\mu}(\cosh \alpha \cosh \beta - 1) + \hat{\mu}^2(\cosh \alpha - \cosh \beta)^2/4]} \\ &\quad + \frac{\rho dr^2}{(\rho - \hat{\mu})} + r^2 d\Omega_{D-2}^2 \\ A_{t,L}^\alpha &= \frac{-n_L^\alpha}{\sqrt{2}} \frac{\hat{\mu} \sinh \alpha [\hat{\mu}(\cosh \alpha - \cosh \beta)/2 + \rho \cosh \beta]}{[\rho^2 + \rho\hat{\mu}(\cosh \alpha \cosh \beta - 1) + \hat{\mu}^2(\cosh \alpha - \cosh \beta)^2/4]} \\ A_{t,R}^\alpha &= \frac{-n_R^\alpha}{\sqrt{2}} \frac{\hat{\mu} \sinh \beta [\hat{\mu}(\cosh \beta - \cosh \alpha)/2 + \rho \cosh \alpha]}{[\rho^2 + \rho\hat{\mu}(\cosh \alpha \cosh \beta - 1) + \hat{\mu}^2(\cosh \alpha - \cosh \beta)^2/4]} \\ M &= \mathbf{1}_k + \begin{pmatrix} P(r)\vec{n}_L\vec{n}_L^T & Q(r)\vec{n}_L\vec{n}_R^T \\ Q(r)\vec{n}_R\vec{n}_L^T & P(r)\vec{n}_R\vec{n}_R^T \end{pmatrix} \end{aligned} \quad (2.6)$$

where

$$\rho \equiv r^{D-3} \quad (2.7)$$

and

$$\begin{aligned} P(r) &= \frac{\hat{\mu}^2 \sinh^2 \alpha \sinh^2 \beta / 2}{[\rho^2 + \rho\hat{\mu}(\cosh \alpha \cosh \beta - 1) + \hat{\mu}^2(\cosh \alpha - \cosh \beta)^2/4]} \\ Q(r) &= \frac{-\sinh \alpha \sinh \beta \hat{\mu}[\rho + \hat{\mu}(\cosh \alpha \cosh \beta - 1)/2]}{[\rho^2 + \rho\hat{\mu}(\cosh \alpha \cosh \beta - 1) + \hat{\mu}^2(\cosh \alpha - \cosh \beta)^2/4]} \quad . \end{aligned} \quad (2.8)$$

These functions P, Q satisfy the relation $P^2 - Q^2 = -2P$.

The mass parameters of these black holes are, in the Einstein and string metric respectively³,

$$\begin{aligned} M_{ADM} &= \widehat{M}_{ADM} \frac{[1 + (D-3) \cosh \alpha \cosh \beta]}{(D-2)} \\ \mu_{string} &= \widehat{\mu} \cosh \alpha \cosh \beta \quad . \end{aligned} \quad (2.9)$$

The electric charges are defined as

$$Q_{(L,R)}^\alpha = \frac{1}{A_{D-2}} \int_{r \rightarrow \infty} r^{D-2} d\Omega_{D-2} F_{rt(L,R)}^\alpha \quad (2.10)$$

and eyeballing the vector potentials we get

$$\begin{aligned} Q_L^\alpha &= \frac{n_L^\alpha}{\sqrt{2}} (D-3) \widehat{\mu} \sinh \alpha \cosh \beta \\ Q_R^\alpha &= \frac{n_R^\alpha}{\sqrt{2}} (D-3) \widehat{\mu} \sinh \beta \cosh \alpha \quad . \end{aligned} \quad (2.11)$$

The dilaton charge

$$\Xi = \frac{1}{A_{D-2}} \int_{r \rightarrow \infty} r^{D-2} d\Omega_{D-2} \nabla_r (\tfrac{1}{2} \Phi) \quad (2.12)$$

is given by

$$\Xi = \frac{(D-3)}{4} \widehat{\mu} (\cosh \alpha \cosh \beta - 1) \quad . \quad (2.13)$$

As they should, all of these parameters reduce to those of the old black hole upon sending the boost parameters α, β to zero.

3 Extremal black holes and supersymmetry

In this Section we will study a particular subclass of the black hole backgrounds obtained above. We will look at the extremal cases which we term “type \mathcal{R} ”⁴ and which are obtained by taking the limit

$$\beta \rightarrow \infty \quad \widehat{\mu} \rightarrow 0 \quad \mu_0 \equiv \tfrac{1}{2} \widehat{\mu} \cosh \beta \quad \text{fixed} \quad . \quad (3.1)$$

³For comparison, the parameter m of [12] is equal to our $\tfrac{1}{2} \widehat{\mu}$ in $D = 4$; note also that our vector field normalizations differ by a factor of two and thus so do the gauge charges.

⁴To avoid confusion with the Type II string, we do not use the terminology “type II” of [12].

There are two other extremal classes which could be considered. The first, type \mathcal{L} , are obtained by switching α for β in the previous equation. This gives extremal black holes with a fixed relationship between mass and left-handed charge, but as we are interested in supersymmetry which lives on the right side of the string, we will not concern ourselves with this class of extremal black holes further. The other class of extremal black holes is a subset of both type \mathcal{R} and type \mathcal{L} , which can be obtained as a subclass of the type \mathcal{R} black holes as follows:

$$\mu_0 \rightarrow 0 \quad \cosh \alpha \rightarrow \infty \quad \mu_1 \equiv \mu_0 \cosh \alpha \quad \text{fixed} \quad . \quad (3.2)$$

The magnitudes of the left- and right-handed charges are equal in this case, and are fixed in relation to the mass. All of these extremal black holes have null singularities, i.e. the horizon coincides with the singularity.

The fields of the type \mathcal{R} black holes are

$$\begin{aligned} dS^2 &= -e^{2\Phi} dt^2 + dr^2 + r^2 d\Omega_{D-2}^2 \\ A_{t,L}^\alpha &= n_L^\alpha \frac{-\rho\sqrt{2}\mu_0 \sinh \alpha}{[\rho^2 + 2\rho\mu_0 \cosh \alpha + \mu_0^2]} \\ A_{t,R}^\alpha &= n_R^\alpha \frac{-(\rho \cosh \alpha + \mu_0)\sqrt{2}\mu_0}{[\rho^2 + 2\rho\mu_0 \cosh \alpha + \mu_0^2]} \\ e^{-2\Phi} &= \frac{1}{\rho^2} [\rho^2 + 2\rho\mu_0 \cosh \alpha + \mu_0^2] \\ M &= \mathbf{1}_k + \begin{pmatrix} P(r)\vec{n}_L\vec{n}_L^T & Q(r)\vec{n}_L\vec{n}_R^T \\ Q(r)\vec{n}_R\vec{n}_L^T & P(r)\vec{n}_R\vec{n}_R^T \end{pmatrix} \end{aligned} \quad (3.3)$$

where

$$\begin{aligned} P(r) &= \frac{2\mu_0^2 \sinh^2 \alpha}{[\rho^2 + 2\rho\mu_0 \cosh \alpha + \mu_0^2]} \\ Q(r) &= \frac{-2\mu_0 \sinh \alpha (\rho + \mu_0 \cosh \alpha)}{[\rho^2 + 2\rho\mu_0 \cosh \alpha + \mu_0^2]} \quad . \end{aligned} \quad (3.4)$$

The charges on the type \mathcal{R} black holes are seen to be

$$\begin{aligned} M_{ADM} &= \frac{A_{D-2}}{8\pi G_N} (D-3) \mu_0 \cosh \alpha \\ Q_L^\alpha &= (D-3) \sqrt{2} \mu_0 \sinh \alpha n_L^\alpha \\ Q_R^\alpha &= (D-3) \sqrt{2} \mu_0 \cosh \alpha n_R^\alpha \end{aligned}$$

$$\Xi = \frac{(D-3)}{2} \mu_0 \cosh \alpha = \frac{1}{2\sqrt{2}} |\vec{Q}_R| \quad . \quad (3.5)$$

From this we find a relation between mass parameter and both charges

$$\frac{\mu_{string}^2}{4 \cosh^2 \alpha} = \mu_0^2 = \frac{1}{2(D-3)^2} [\vec{Q}_R^2 - \vec{Q}_L^2] \quad (3.6)$$

and a relation between the ADM mass and right-handed charge

$$M_{ADM}^2 = \left[\frac{A_{D-2}}{16\pi G_N} \right]^2 2\vec{Q}_R^2 \quad (3.7)$$

This last formula looks suspiciously like a supersymmetry condition.

We would now like to find out whether the type \mathcal{R} black holes do indeed possess a supersymmetry. Supersymmetry of four-dimensional electric-magnetic black holes with Kaluza-Klein charges has been established previously in [22].

It is easiest to start with the transformation rules written in the sigma-model frame where they are the simplest. One can obtain the transformations in the Einstein variables by doing the appropriate field redefinitions. The ten dimensional supersymmetry transformations are, for the gravitino, dilatino and gaugino respectively,

$$\begin{aligned} \delta \tilde{\psi}_m &= \tilde{\nabla}_m \tilde{\epsilon} - \frac{1}{8} \tilde{H}_{mnp} \tilde{\Gamma}^{np} \tilde{\epsilon} \\ \delta \tilde{\chi} &= \frac{-1}{2} \tilde{\Gamma}^m \tilde{\partial}_m \tilde{\Phi} \tilde{\epsilon} + \frac{1}{12} \tilde{H}_{mnp} \tilde{\Gamma}^{mnp} \tilde{\epsilon} \\ \delta \tilde{\lambda}^I &= \frac{1}{2} \tilde{F}_{mn}^I \tilde{\Gamma}^{mn} \tilde{\epsilon} \end{aligned} \quad (3.8)$$

where \tilde{F}^I are the gauge fields of the Yang-Mills multiplet. The supersymmetry variations of the bosonic fields, which are proportional to the fermionic fields, are of course zero because we are considering a bosonic background of the theory.

Using the ten dimensional supersymmetry rules (3.8) and the dictionary of [14, 18, 19, 20] to relate ten and D dimensional fields we obtain the supersymmetry transformations for the toroidally compactified theory in any dimension D . Firstly, we note that the p internal gravitinos and 16 gauginos will combine to make $(16+p)$ modulinos, which is precisely the correct number appropriate for the number of bosonic moduli [23]. The next step is to untangle the relations between the moduli matrix and the internal components of the ten dimensional metric, antisymmetric tensor, and gauge fields, using the basis change (1.22),(1.23) and the ten versus D dimensional field relations (1.25). The fields so obtained are then used to

get the supersymmetry variations of the fermions in the toroidally compactified “ D dimensional” bosonic black hole background in their full glory. We will list here the fields which are nonzero for type \mathcal{R} black hole backgrounds.

Defining $\tau \equiv \sum_{a=1}^p n_R^a n_L^a$ and $\delta \equiv 1 - \sum_{I=1}^{16} n_L^I n_L^I$, we find that

$$\begin{aligned}
e^{\tilde{\Phi}} &= \frac{\rho}{R(\rho)} \\
\tilde{A}_t^I &= \frac{\sqrt{2}\mu_0 \sinh \alpha}{R(\rho)} n_L^I \\
\tilde{A}_\alpha^I &= \frac{-\sqrt{2}\mu_0 \sinh \alpha}{R(\rho)} n_L^I n_R^\alpha \\
\tilde{B}_{\alpha\beta} &= \frac{-\mu_0 \sinh \alpha}{R(\rho)} (n_R^\alpha n_L^\beta - n_L^\alpha n_R^\beta) \\
\tilde{B}_{t\beta} &= \frac{(\mu_0 \cosh \alpha n_R^\beta + \mu_0 \sinh \alpha n_L^\beta)}{R(\rho)} \\
\tilde{G}_{\alpha\beta} &= \delta_{\alpha\beta} + (\delta - 1) \frac{\mu_0^2 \sinh^2 \alpha}{R^2(r)} (n_R^\alpha n_R^\beta) + \frac{-\mu_0 \sinh \alpha}{R(\rho)} (n_R^\alpha n_L^\beta + n_L^\alpha n_R^\beta) \\
\tilde{G}^{\alpha\beta} &= \delta_{\alpha\beta} + \frac{P(r)}{2} (n_R^\alpha n_R^\beta + n_L^\alpha n_L^\beta) + \frac{-Q(r)}{2} (n_R^\alpha n_L^\beta + n_L^\alpha n_R^\beta) \\
\tilde{E}_a^\alpha &= \delta_a^\alpha + \left[-1 + \frac{-Q(r)}{\sqrt{2P(r)}} \right] n_R^\alpha n_R^a + \sqrt{\frac{P(r)}{2}} n_L^\alpha n_R^a \\
\tilde{E}_\alpha^a &= \delta_\alpha^a + \frac{\mu_0 \sinh \alpha}{R(\rho)} \left[\left(-1 + \frac{-Q(r)}{\sqrt{2P(r)}} \right) n_R^\alpha n_R^a + \sqrt{\frac{P(r)}{2}} n_R^\alpha n_L^a \right] \\
A_t^{[1]\alpha} &= A_L n_L^\alpha - A_R n_R^\alpha \quad A_{\alpha t}^{[2]} = A_L n_L^\alpha + A_R n_R^\alpha \quad A_t^{[3]I} = \sqrt{2} A_L n_L^I
\end{aligned} \tag{3.9}$$

where

$$A_L = \frac{\rho \mu_0 \sinh \alpha}{K(\rho)} \quad A_R = \frac{\mu_0 (\rho \cosh \alpha + \mu_0)}{K(\rho)} \tag{3.10}$$

where we have defined

$$\begin{aligned}
R(\rho) &= \rho + \mu_0 (\cosh \alpha + \tau \sinh \alpha) \\
K(\rho) &= \rho^2 + 2\rho\mu_0 \cosh \alpha + \mu_0^2 \quad .
\end{aligned} \tag{3.11}$$

Note that $e^{-\tilde{\Phi}}$ is a harmonic function, although the D dimensional $e^{-\Phi}$ is *not* harmonic unless $\alpha = 0$.⁵

The supersymmetry variations of the fermions become

$$\begin{aligned}
\delta\lambda^I &= (\partial_i\rho)\tilde{\Gamma}^{0i}\left[e^{-\Phi}\tilde{F}_{t\rho}^I(1+n_R^\alpha A_t^{[1]\alpha})+\tilde{\Gamma}^{0a}n_R^\alpha E_a^\alpha\tilde{F}_{t\rho}^I\right]\tilde{\epsilon} \\
\delta\tilde{\psi}_\alpha &= \frac{1}{4}(\partial_i\rho)e^{-\Phi}\tilde{\Gamma}^{0i}\left[\left(-G_{\alpha\beta}F_{t\rho}^{[1]\beta}-\tilde{H}_{\alpha t\rho}+A_t^{[1]\beta}\tilde{H}_{\alpha\beta\rho}\right)\right. \\
&\quad \left.-\tilde{\Gamma}^{0b}E_b^\beta e^{-\Phi}\left(\partial_\rho G_{\alpha\beta}-\tilde{H}_{\alpha\beta\rho}\right)\right]\tilde{\epsilon} \\
\delta\tilde{\psi}_0 &= (\tfrac{1}{2}\partial_i\rho)\tilde{\Gamma}^{0i}\left[-\partial_\rho\Phi+\tfrac{1}{2}E_a^\alpha e^{-\Phi}\tilde{\Gamma}^{0a}\left(G_{\alpha\beta}F_{t\rho}^{[1]\beta}-\tilde{H}_{\alpha t\rho}+A_t^{[1]\beta}\tilde{H}_{\alpha\beta\rho}\right)\right]\tilde{\epsilon} \\
\delta\chi &= (\tfrac{1}{2}\partial_i\rho)\tilde{\Gamma}^i\left[-\partial_\rho\tilde{\Phi}+e^{-\Phi}E_a^\alpha\tilde{H}_{t\alpha\rho}\tilde{\Gamma}^{0a}+\tfrac{1}{2}E_a^\alpha E_b^\beta\tilde{H}_{\rho\alpha\beta}\tilde{\Gamma}^{ab}\right]\tilde{\epsilon} \\
\delta\tilde{\psi}_i &= \partial_i\tilde{\epsilon}+(\tfrac{1}{4}\partial_i\rho)E_a^\alpha e^{-\Phi}\left[-G_{\alpha\beta}F_{t\rho}^{[1]\beta}+\tilde{H}_{\alpha t\rho}-A_t^{[1]\beta}\tilde{H}_{\alpha\beta\rho}\right]\tilde{\Gamma}^{0a}\tilde{\epsilon} \\
&\quad +(\tfrac{1}{8}\partial_i\rho)\left[(E_a^\alpha\partial_\rho E_{\alpha b}-E_b^\alpha\partial_\rho E_{\alpha a})-E_a^\alpha E_b^\beta\tilde{H}_{\alpha\beta\rho}\right]\tilde{\Gamma}^{ab}\tilde{\epsilon}
\end{aligned} \tag{3.12}$$

and using the fields (3.9) we get

$$\begin{aligned}
\delta\tilde{\lambda}^I &= (\partial_i\rho)n_L^I\tilde{\Gamma}^{0i}\frac{\sqrt{2}\mu_0\sinh\alpha}{R(\rho)\sqrt{K(\rho)}}\left[1-n_R^a\tilde{\Gamma}_{0a}\right]\tilde{\epsilon} \\
\delta\tilde{\psi}_\alpha &= (\tfrac{1}{2}\partial_i\rho)\tilde{\Gamma}^{0i}\frac{\mu_0\sinh\alpha}{R(\rho)\sqrt{K(\rho)}}\left(n_L^\alpha+\frac{\mu_0\sinh\alpha(1-\delta)}{R(\rho)}n_R^\alpha\right)\left[1-n_R^a\tilde{\Gamma}_{0a}\right]\tilde{\epsilon} \\
\delta\psi_0 &= (\tfrac{1}{2}\partial_i\rho)\tilde{\Gamma}^{0i}\frac{\mu_0(\rho\cosh\alpha+\mu_0)}{\rho K(\rho)}\left[1-n_R^a\tilde{\Gamma}_{0a}\right]\tilde{\epsilon} \\
\delta\tilde{\chi} &= (\tfrac{1}{2}\partial_i\rho)\tilde{\Gamma}^i\left(\frac{-\mu_0(\cosh\alpha+\tau\sinh\alpha)}{\rho R(\rho)}+\frac{-n_L^b\tilde{\Gamma}^{0b}\mu_0\sinh\alpha}{R(\rho)\sqrt{K(\rho)}}\right)\left[1-n_R^a\tilde{\Gamma}_{0a}\right]\tilde{\epsilon} \\
\delta\tilde{\psi}_i &= \partial_i\tilde{\epsilon}-\tfrac{1}{2}\partial_i\Phi\tilde{\Gamma}_{0a}n_R^a\tilde{\epsilon} \quad .
\end{aligned} \tag{3.13}$$

All but one of these equations may be satisfied by requiring that $\tilde{\epsilon}$ satisfy the algebraic

⁵This makes the form of multi-black hole solutions hard to guess, and is a symptom of the fact that the D dimensional dilaton and gauge fields for a multi-black hole are in fact built out of more than one harmonic function (see also [31, 30]).

projection condition

$$\left[1 - n_R^a \tilde{\Gamma}_{0a}\right] \tilde{\epsilon} = 0 \quad . \quad (3.14)$$

Notice that this condition involves only the right-handed vector \vec{n}_R , which makes sense as the supersymmetry lives on the right side of the string.

Lastly, the projection condition (3.14) yields

$$\delta\tilde{\psi}_i = \partial_i \tilde{\epsilon} - \frac{1}{2} \partial_i \Phi \tilde{\epsilon} \quad (3.15)$$

which can be satisfied if

$$\tilde{\epsilon} = e^{\Phi/2} \tilde{\epsilon}^{(0)} \quad (3.16)$$

where $\tilde{\epsilon}^{(0)}$ is a constant spinor satisfying (3.14). This $\tilde{\epsilon}$ is the spinor appropriate to the string metric, and it is independent of the time and the internal coordinates, (t, x^α) , and of \vec{n}_L . It depends on the vector \vec{n}_R only through the projection condition (3.14).

The number of unbroken supersymmetry parameters is determined by (3.14). We proceed by defining the quantities

$$\tilde{P}^\pm(\vec{n}_R) = \frac{1}{2} \left[\mathbf{1} \pm \tilde{\Gamma}_0 n_R^a \tilde{\Gamma}_a \right] \quad (3.17)$$

which can easily be seen to be projectors by using the Dirac algebra and the fact that \vec{n}_R is a unit vector. In addition $\tilde{P}^\pm(\vec{n}_R)$ commute with $\tilde{\Gamma}_{11}$. We form the new [chiral] spinor combinations

$$\tilde{\epsilon}^\pm(\vec{n}_R) = \tilde{P}^\pm(\vec{n}_R) \tilde{\epsilon} \quad (3.18)$$

and note that supersymmetry then requires via the projection condition (3.14) that the $\tilde{\epsilon}^-$ vanish. In the meantime, the $\tilde{\epsilon}^+$ are unconstrained, so we see that the type \mathcal{R} extremal black hole backgrounds break *half* of the ten dimensional supersymmetries, leaving eight unbroken supersymmetry parameters.

The number of unbroken supersymmetries in D dimensions depends on how the ten dimensional spinors break up into D dimensional spinors. We take our cue for which extended supersymmetry is appropriate in a given D by matching the coset $O(16+p, p)/[O(16+p) \otimes O(p)]$, which parametrizes the scalar manifold, to those found for various supergravities in [23]. We find that the appropriate theory is $N = 4$ in $D = 4, 5$; $N = 2$ in $D = 6, 7$; and $N = 1$ in $D = 8, 9$. In each case, the group $O(p)$ is the automorphism group for the supersymmetry algebra; respectively these are $SU(4), USp(4), SU(2) \otimes SU(2), USp(2), U(1), \mathbf{1}$ [23]. In $D = 6$ the two $SU(2)$'s reflect the chiral nature of the supersymmetry algebra. We mention here

that, despite appearances, it is possible to have a central charge in the asymptotic supersymmetry algebra even for the cases $D = 8, 9$ where the supersymmetry is only $N = 1$ [24]. (This happens because the spinors can be taken to be [pseudo-]Majorana, and the Γ^0 matrix is symmetric in this representation.) For the other cases there are $[\frac{N}{2}]$ central charges.

For the case $D = 4$ an explicit representation of the $\tilde{\Gamma}$ matrices is

$$(\tilde{\Gamma}^m, \tilde{\Gamma}^a) = (\Gamma^m \otimes \mathbf{1}, \Gamma_5 \otimes \Pi^a) \quad (3.19)$$

where $(\Gamma_5)^2 = \mathbf{1}$. With a split of $\tilde{\epsilon} = \epsilon \otimes \eta \equiv (\epsilon^A)$, where A runs from $1 \dots N = 4$, the projectors are $P^{\pm A}_B = \frac{1}{2}(\mathbf{1}_B \pm \Gamma_0 \Gamma_5 n^a_R \Pi^A_{aB})$. We therefore get $N = 2$ supersymmetry, with the central charges equal and proportional to the magnitude of the electric charge. We note that this agrees with the analysis of [25] for the appropriate $D = 4$ single gauge field subclass of the black holes presented here.

To summarize, we have found that *all* of the type \mathcal{R} black holes in dimension $D = 4 \dots 9$ preserve half of the supersymmetries. We expect that this will protect the relation between the mass and the right-handed charge in all dimensions $D = 4 \dots 9$.

4 Entropy and the stretched horizon

Here we will investigate the entropy and the stretched horizon of extremal type \mathcal{R} black holes. We notice firstly that the event horizon of a generic black hole of Section 2 is located where $\rho = r^{D-3} = \hat{\mu}$, i.e.

$$r_H = \hat{\mu}^{\frac{1}{D-3}} \quad (4.1)$$

To calculate the Hawking temperature, we change variables to u, Θ , defined by

$$\begin{aligned} u &= \frac{2}{(D-3)} \sqrt{r^{D-3} - r_H^{D-3}} \\ \Theta &= \frac{i(D-3)t}{r_H(\cosh \alpha + \cosh \beta)} \end{aligned} \quad (4.2)$$

We obtain for the near-horizon string metric⁶ in (u, Θ) variables

$$dS^2 \sim r_H^{5-D} (u^2 d\Theta^2 + du^2) + r_H^2 d\Omega_{D-2}^2 \quad (4.3)$$

⁶The conversion factor at the horizon from the string metric to the Einstein metric is simply a constant, $\Gamma_H^{\gamma/2}$, where $\sqrt{\Gamma_H} = \frac{1}{2}(\cosh \alpha + \cosh \beta)$.

which is Rindler space in the (u, Θ) directions. We see from the definition of Θ above, and the fact that Θ must be 2π periodic, that the Hawking temperature is given by

$$T = \frac{(D-3)}{2\pi r_H (\cosh \alpha + \cosh \beta)} \quad . \quad (4.4)$$

In the type \mathcal{R} extremal limit, we see that the temperature (4.4) goes to zero, except for the special case $D = 4$ where it remains finite due to a scaling “accident” (see equation (3.1)). However, this finite temperature in $D = 4$ may be unphysical, in view of the supersymmetry properties derived in the previous Section (see also [25]). We will take the temperature of all of the type \mathcal{R} black holes to be zero.

Let us next inspect the fields of these black hole backgrounds for their behavior near the horizon. We keep the Newton constant fixed in our discussions; upon restoring the factors of the string coupling at asymptotic infinity we have for the string metric

$$dS^2 = \frac{-\rho^2 g_\infty^{2\gamma} dt^2}{[\rho^2 + 2\rho\mu_0 \cosh \alpha + \mu_0^2]} + g_\infty^{2\gamma} [dr^2 + r^2 d\Omega_{D-2}^2] \quad (4.5)$$

so for extremal black holes

$$\bar{r} = g_\infty^\gamma r \quad (4.6)$$

measures proper distance from the horizon, in string metric.

On the horizon, we find that

$$\begin{aligned} R_{tt} &\sim \bar{r}^{2(D-4)} \\ R &\sim \frac{1}{\bar{r}^2} \\ e^{-2\Phi} &\sim \frac{\mu_0^2}{\rho^2} \\ F_{rt,L}^\alpha &\sim -Q_L^\alpha \frac{r^{D-4}}{\mu_0^2} \\ F_{rt,R}^\alpha &\sim +Q_R^\alpha \frac{r^{D-4}}{\mu_0^2} \quad . \end{aligned} \quad (4.7)$$

We see that at string proper distance

$$\bar{r} = C \quad \text{i.e.} \quad r = \frac{C}{g_\infty^\gamma} \quad (4.8)$$

where C is a pure number of order one, higher order corrections to the action (1.3) will become important. This is the location of the *stringy* stretched horizon, and it is independent of any of the parameters of the black hole.

Now we come to the entropy. In order to calculate it for a black hole, we use the method of Gibbons and Hawking, working along similar lines to [28]. By examination of the field equations corresponding to the action in the Einstein metric, (1.29), we find that on shell the scalar curvature R_g cancels against the scalars Φ, M . We also find that the on-shell action, including the necessary extrinsic curvature term and with the numerical coefficient reinstated, can be written as a surface term:

$$S = \frac{1}{8\pi G_N} \int_{\partial M} \left[(K - K_0) + \frac{1}{4} n_\mu \left(\sqrt{-g} e^{-\gamma\Phi} A_\nu^\alpha (LML)_{\alpha\beta} F^{\beta\mu\nu} \right) \right] \quad . \quad (4.9)$$

We obtain for the entropy the expected relation

$$\Sigma = \frac{1}{4G_N} R_E^{D-2} A_{D-2} \quad (4.10)$$

where $G_N = M_{Pl}^{2-D}$ is the Newton constant and R_E is the radius of the local $(D-2)$ -sphere in the Einstein metric.

For the extremal black holes which we are studying, the singularity is null and the area of the event horizon is classically zero. However, we do not expect this zero area to survive higher order corrections. In other words, we will use the entropy calculated at the *stringy* stretched horizon, which is located at of order one string unit of proper distance away from the event horizon as in (4.8). With use of the relation (1.27) between the string and Einstein metrics, the Einstein radial variable is found to be

$$R_E^{D-2}(D) = [r_{S.H.}] \mu_0 \quad . \quad (4.11)$$

The entropy for the type \mathcal{R} extremal black holes, calculated at the stringy stretched horizon, is thus

$$\Sigma_{S.H.} = \frac{A_{D-2}}{4G_N} \frac{C}{g_\infty^\gamma} \mu_0 \quad . \quad (4.12)$$

In units where $\langle e^\Phi \rangle_\infty = g_\infty^2$, electric charges scale as

$$\vec{Q}(g_\infty = 1) = \frac{1}{g_\infty^\gamma} \vec{Q}(g_\infty \neq 1) \quad (4.13)$$

but μ_0 stays fixed as it is proportional to the ADM mass which stays fixed with G_N . The dilaton charge does not scale either. Using the relation (3.6) between the mass parameter and the left and right charges

$$\mu_0 = \frac{1}{g_\infty^\gamma} \frac{1}{\sqrt{2}(D-3)} \sqrt{\vec{Q}_R^2 - \vec{Q}_L^2} \quad (4.14)$$

we obtain finally

$$\Sigma_{S.H.} = \frac{1}{g_\infty^{2\gamma}} \left[\frac{C A_{D-2}}{4\sqrt{2}G_N(D-3)} \right] \sqrt{\vec{Q}_R^2 - \vec{Q}_L^2} \quad . \quad (4.15)$$

We might ask at this stage what would happen if we were to take into account quantum corrections to the fields. It has been argued previously [8] that quantum corrections would lead only to a renormalization of the coefficient in front of the $O(16+p, p)$ -invariant combination of the charges. In general, the tree level action will be modified by higher order corrections, which will lead to corrections to the solution and to the supersymmetry transformations. The $O(16+p, p)$ symmetry and the solution-generating symmetry are expected to be valid to all orders in α' (see e.g. [17]), but the transformation laws to a given order may look different to the tree level laws when written in terms of the fields like $G_{\mu\nu}$ which appear in the low-energy effective action. Therefore, in general, both of the quantities which feed into the entropy, viz. the coefficient of $d\Omega_{D-2}^2$ and the form of $e^{-2\Phi}$ near the horizon, will be modified. We expect by dimension counting and $O(16+p, p)$ invariance (which is available in some scheme) that the area term for the entropy will be renormalized by a factor changing only the number C/G_N . We will absorb this factor into C . Note that, besides the area term, there will also be subleading corrections to the area term for the entropy to which our calculations are not sensitive.

If we choose to keep the string scale fixed, and not the Newton constant as we have done here, then we find that the black hole entropy is proportional to $1/g_\infty^2$ in *all* dimensions D . This result for the entropy was explained [for a non-extremal black hole] in $D = 4$ in terms of genus zero stringy configurations in [1, 11]. There it was also found that, in the limit of a very large mass black hole (where the subleading terms are not present), the area term for the entropy would receive corrections from higher genera which would serve only to renormalize the Newton constant.

The next thing to calculate is the logarithm of the degeneracy of the elementary (electrically charged) string state appropriate to the type \mathcal{R} black hole. This corresponds to the entropy of a string state with given mass and charges. The degeneracy corresponds, of course, to the different possible ways of distributing oscillators.

From the mass relations on the left and right sides of the heterotic string, and the fact that the moduli matrix tends asymptotically to the identity matrix, we know that

$$m^2 = g_\infty^2 \left[\frac{\vec{Q}_R^2}{g_\infty^{4\gamma}} + 2(N_R - \tfrac{1}{2}) \right] = g_\infty^2 \left[\frac{\vec{Q}_L^2}{g_\infty^{4\gamma}} + 2(N_L - 1) \right] \quad (4.16)$$

In this formula, the mass of the string state is measured in Einstein frame (in Planck units) so as to facilitate comparison to the black hole mass; this gives rise to the overall factor of g_∞^2 . The factors of $g_\infty^{2\gamma}$ under the charges Q arise because the field equations tell us that the conserved charges are “ $Q/g_\infty^{2\gamma}$ ”. (Note that there are no α' 's around; these have all been converted into powers of g_∞ and of G_N which is fixed, via the relation $m_{Pl}^{-2} = G_N^\gamma = g^2 \alpha'$.)

The left and right charges are then related by

$$2(N_L - 1) - 2(N_R - \tfrac{1}{2}) = \frac{1}{g_\infty^{4\gamma}} \left[\vec{Q}_R^2 - \vec{Q}_L^2 \right] \quad (4.17)$$

where N_L, N_R are the contributions to the mass from the oscillators excited on the left and right sides of the string. For the electrically charged states which would correspond to the type \mathcal{R} black holes, supersymmetry demands that the right-handed sector be in the ground state, i.e. $N_R = \frac{1}{2}$.

The level density d_{ES} of elementary string states as a function of N_L is a standard formula; it goes at large N_L as

$$d_{ES} \sim (\text{power prefactors}) e^{\sqrt{N_L}/T_H} \quad (4.18)$$

where $T_H = 1/(4\pi)$ is the Hagedorn temperature. So we find that at large N_L the leading term is

$$\log(d_{ES}) \sim \frac{1}{g_\infty^{2\gamma}} \frac{1}{\sqrt{2}T_H} \sqrt{\vec{Q}_R^2 - \vec{Q}_L^2} \quad . \quad (4.19)$$

Putting together the black hole and string entropies we obtain the result

$$\Sigma_{S.H.} = \left[\frac{CT_H^{(0)} A_{D-2}}{4G_N(D-3)} \right] \log(d_{ES}) \quad . \quad (4.20)$$

Note in particular that the constant of proportionality between the black hole entropy calculated at the stretched horizon and the logarithm of the degeneracy of the elementary string states is independent of the string coupling. This is a direct consequence of the fact that we took the stretched horizon to be one *string* unit of proper distance away from the event

horizon. If we wish to set the two entropies equal to each other, then the numerical factor can be arranged with $C \sim 1$.

To finish, we note that in making the identification of the supersymmetric black hole and string state entropies, we had *no* freedom to soak up any factors of the mass by using a redshift factor. This is a phenomenon peculiar to extremal black holes, and has its origins in the balance of forces which permits the supersymmetry. The static force between two of these objects is zero, due to cancellation of attractive gravitational and scalar forces by repulsive electromagnetic forces. Thus any redshift, which can also be thought of as a gravitational dressing, would be cancelled out by dressings from electromagnetic and scalar effects.

5 Conclusion

In this paper we have studied the D dimensional generalizations of the electrically charged black holes of [8]. We have found that the entropy of a type \mathcal{R} extremal black hole with a given mass and charges, when calculated at the stringy stretched horizon, is the same as the entropy of the corresponding string state, up to a numerical factor of order 1⁷. That this correspondence works in dimensions other than four is a satisfying consistency check. A remaining puzzle is to understand the precise nature of the stringy physics which leads to the existence of the stretched horizon for extremal black holes.

We have also seen that the type \mathcal{R} extremal black holes are supersymmetric, as was expected from the fact that they are extremal and nonrotating and also from the conjectured correspondence to string states. The structure of the unbroken target space supersymmetries, involving the projection condition (3.14) is an explicit example of a general phenomenon found by Kallosh recently[31]. The fact that half of the supersymmetries are unbroken is a result of the fact that there is only electric charge. If there were also magnetic charge in four dimensions, then we expect that only one-quarter of the supersymmetries would remain unbroken[25, 22].

We regard these findings as additional support for the idea that extremal black holes of toroidally compactified heterotic string theory may be thought of as string states of the same theory. Another consistency check to perform is to compare the results of tree level scattering of the elementary states in string theory with a semiclassical moduli space calculation of the same process for the (extremal) black holes[29].

Some of the previous comparisons of the entropy of black holes and string states have concentrated on nonextremal black holes. These would correspond to string states with mass

⁷and up to subleading corrections which we have not addressed here

greater than charge. Both of these objects are unstable; one because of Hawking radiation and the other because they would correspond to ten dimensional massive states which are apt to decay (see, e.g. [32]). Comparison of these two process is likely to be difficult, not least due to problems in extracting the widths of excited states which are not on the leading Regge trajectory in string theory[33]. However, it would be interesting to know if these two processes are related somehow.

While this work was in progress we were informed by M. Cvetič that supersymmetry of four dimensional extremal dyonic black holes of toroidally compactified heterotic string theory has been proven very recently in [34].

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Appendix

Here we list our notation and conventions.

Indices from the first part of the alphabet are internal (the toroidal directions), and those from the last part are external. We use small greek letters for curved indices, and small latin letters for tangent space indices. For the time coordinate, we use t for a curved index and 0 for a tangent space index. Where a distinction for gauge indices is useful, capital latin indices are reserved for “true” gauge indices, indicating charges or fields from the ten dimensional Yang–Mills multiplet. All repeated indices are summed over.

The spacetime signature is “mostly plus”, $(-, +, \dots, +)$. Antisymmetric combinations of gamma matrices are $\Gamma^{[m_1, \dots, m_n]} = \Gamma^{m_1} \dots \Gamma^{m_n}$, and antisymmetrization is done with weight one, e.g. $\Gamma^{mn} = \frac{1}{2}(\Gamma^m \Gamma^n - \Gamma^n \Gamma^m)$.

Covariant derivatives on spinors are given by $\nabla_m \epsilon = \partial_m \epsilon + \frac{1}{4} \omega_{mnp} \Gamma^{np} \epsilon$. Any derivative acts only on the object sitting immediately to its right.

Bars on fields denote the special combinations (1.8)–(1.9) which transform simply under

the solution-generating transformation of Section 1. Hats denote the fields of the original rotating uncharged black hole which was used to generate electrically charged backgrounds. Ten dimensional fields are denoted by tildes.

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